Why is COVID-19 Mortality in Lombardy so High? Evidence from the Simulation of a SEIRHC Model*

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Abstract

The standard SEIR model based on a parameterization consistent with the international evidence cannot explain the very high COVID-19 related mortality in Lombardy. This paper proposes an extension of the standard SEIR model that is capable of solving this puzzle. The SEIR model features exogenous mortality: once Susceptible individuals become first Espoused, and then Infected they succumb with a given probability. Our extended model takes into account the Hospitalization process and the possibility that Hospitalized patients, who need to resort to Intensive Care Unit, cannot find availability because the ICU is saturated. This Constraint creates an additional increase in mortality which is endogenous to the diffusion of the disease. The extended SEIRHC (H stands for Hospitalization and C stands for Constraint) is capable of explaining the dynamics of COVID-19 related mortality in Lombardy with a parameterization consistent with the international evidence.

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1 Introduction

COVID-19 related mortality in Lombardy is way above all international evidence. Using data made available by the Civil Protection, https://github.com/pcm-dpc/COVID-19, Figure 1 illustrates the ratio of Fatalities (morti) to total cases (casi_totali) observed daily over the period February 24th 2020-March 31st 2020 which shows a reported lethality growing rather steadily from an initial 4 per cent to a value reaching 16 per cent at the end of the sample (6818 Fatalities for 42161 total cases), the upward trend in mortality is paired with a descending trend in the ratio of patients hospitalized in ICU to total hospitalized patients. This pattern of lethality cannot be replicated by standard SEIR model, which has been successfully applied to the analysis of COVID-19 diffusion in China [Wu, et. al, Kucharski et. al.]. The third line in the graph reports the pattern of the ratio of fatalities to the sum of Exposed, Recovered and Removed as Fatalities generated by a SEIR model with an internationally consistent CFR of 0.138 (see, for example, Verity et al(2020)) which is nowhere near the observed data.

Two possible explanations can be considered for a discrepancy between observed data and model simulated data: either the model is wrong or the data are wrong. As matter of fact, the data can be wrong the total observed cases observe do not include patients with mild symptoms, which were not hospitalized and were therefore not tested.

In this paper we explore the possibility that the standard model misses an important dimension that is instead reflected in the data.

Figure 2 that reports daily fatalities and the number of patients in ICU, illustrates that, despite a remarkable effort to expand the number of ICU daily fatalities have been rather steadily increasing over time.

INSERT FIGURES 1-2 HERE

The mismatch between data and model prediction can be related to a specific feature of the SEIR model: mortality is exogenously given by a constant parameter. In the SEIR specification Infectious patients are divided in three groups, Mild, Severe and Fatal, and the destiny of each patient is written at the time they are exposed to the disease: after some heterogeneous period Mild and Severe patients inevitably recover while fatal patients inevitably die. What if what happened in Lombardy can be described as follows: infectious patients are still divided in the three standard groups, but the Severe do not inevitably recover. In fact, some of them need assistance in Intensive Care Unit and if there are no available ICU positions, then their status changes from Severe to Fatal. There is therefore a time-varying endogenous component of mortality that cannot be captured by the standard SEIR model.
This paper extends the SEIR model to a SEIRCH that follows patients in their pattern of Hospitalization and endogeneize the possible Constraint of ICU availability.

The model is calibrated to the data from Lombardy to illustrate that it is capable to replicate the observed pattern of lethality in Lombardy with a parametrization fully in line with the international evidence.

2 The SEIRCH Model: Description

The SEIRCH model is a system of differential equations for the dynamics of a virus across different groups of the population.

The exact specification of the equations is reported in the Appendix, Figure 3 reports the Dependency Graph of the Model, while this section describes its structure and fundamental elements.

The model allows to simulate the dynamics of the virus diffusion starting from an initial period in which the total Population ($N_t$) of $N$ individuals is divided in 1 Infectious ($I_t$) and $N-1$ Susceptible ($S_t$). In each period (day) some Susceptible become Exposed ($E_t$), their number is determined by the basic reproduction number $R_0$, that determines the number of secondary infections each infected individual produces, by the probability with which Susceptible meets Infectious, $\left(\frac{I_{t-1}}{N_{t-1}}\right)$, and by the average duration of the period in which a patient is infectious $T_{inf}$. Exposed after an incubation period of length $T_{inc}$, become Infectious. The outflows from Susceptible is the inflows into Exposed in each period, and the outflows from Exposed is the inflows into Infectious. Infectious falls into three groups: those with mild symptoms ($MILD_t$), those with severe symptoms ($SEV_t$), and those with fatal symptoms ($FAT_t$). The allocation to these groups is controlled by three probabilities: $(1 - p^{sev} - p^{fat})$, $p^{sev}$, $p^{fat}$. Patients with mild symptoms recover after a recovery period, $T_{srec}$. Patients with severe and fatal symptoms require hospitalization, both these group are hospitalized after a period between developing symptoms and hospitalization of average duration $T_{shosp}$, hospitalized patients require intensive care unit with probability $p^{ic}$. Patients with fatal symptoms succumb notwithstanding hospitalization, even in intensive care, after the mean duration from the onset of symptoms to death, $T_{sd}$. Patients with severe symptoms either recover or become fatal. The recovered, with a mean duration of from the onset of symptoms to hospital discharge of $T_{shd}$, are those who do need intensive care unit and those who need intensive care unit and find a
place. The patients with severe symptoms that need ICU and do not find availability become fatal. At the end of each period the population decreases because of the fatalities, while the stock of recovered grows as a consequence from the new additions of recovered with mild and severe symptoms.

After calibration, we shall compare model simulated data with observed data from Lombardy to assess the potential of the model and its explanation of the mortality in Lombardy.

3 The SEIRCH Model: a Calibration to data from Lombardy

Model simulation requires numerical values for all the relevant parameters. Given the availability of a sample of sufficient size of reliable data, parameters can be estimated (see Cereda et al.(2020)). The daily data made available on Lombardy by Protezione Civile cover a short sample daily sample of about forty observations and are affected by a change in regime as on March 8 2020 a full lockdown was legislated for the region and the entire country.

Measurement error in the data has strong implications for the estimation of the crucial model parameters and for the design of optimal policies for them (see Stock(2020)).

These considerations led to the design and implementation of a calibration strategy that allowed to select the dating of the lockdown in the sample of simulated data and to estimate the impact of the lockdown on the basic reproduction number $R_0$. $R_0$ was set initially at 2.2 in line with the international evidence, reflected in the baseline parameterization in the epidemic calculator available online (https://gabgoh.github.io/COVID/index.html).

Also all the other parameters that determines the transmission dynamics and the clinical dynamics were chosen in line with the international evidence. The calibration of these parameters is summarized in Table 1.

\begin{center}
\textbf{INSERT TABLE 1 HERE}
\end{center}

To set the value of $R_0$ after the intervention the model was simulated first in a pre-lockdown scenario, when the capacity constraint in terms of ICU was still irrelevant, with an initial population of 10 millions, and all duration parameters set in line with the international evidence. The lockdown was then dated in the model simulated data to match the number of daily fatalities observed on the March 8 2020. With this procedure we dated March 8th as day 95 of our 730 (2 years) of simulated
Having dated the lockdown, the post lockdown $R_0$ was calibrated to match the observed number of hospitalized patients two weeks after the lockdown. This procedure delivered a $R_0$ post lockdown of 0.95.

Finally, the probability with which of an hospitalized patient needs intensive care was calibrated at 0.16 which coincides with the maximum of the observed ratio of COVID patients in ICU to total hospitalized COVID patients in Lombardy after the lockdown.

### 4 The SEIRCH Model: Simulations

The model has been simulated under two scenarios. In the baseline scenario the IC capacity is set to 400 beds before the lockdown (the maximum of the observed utilization) and to the observed number of ICU beds until ICU utilization has reached its peak at 1330 beds, from then onwards it has been kept constant at that level. In the alternative scenario the capacity has been set at 3000 units, a value for which the constraint would always be non-binding in the simulation.

Figure 4 reports the pattern of total hospitalized patients and observed hospitalization. Total hospitalization is the same under the two scenarios and the pattern of the data is tracked by the model, simulated data place the peak of hospitalization at mid-April 2020.

**INSERT FIGURE 4 HERE**

The results of crucial for the model extension is reported in Figure 5. Figure 5 clearly illustrates that the capacity constraint is essential to replicate the pattern of mortality observed in Lombardia, the model without the ICU capacity constraint gets nowhere near explaining the observed pattern of observed mortality, while the model with the capacity constraint included gets very close to the observed data.

**INSERT FIGURE 5 HERE**

The expansion of ICU capacity has been crucial to save lives in Lombardy.

Figure 6 reports the pattern of model simulated total recovered, model simulated hospital recovered and patients labelled recovered (guar- iti).

**INSERT FIGURE 6 HERE**

The model based variables that tracks well the observed recovered patient is the patients recovered from hospital while the effective number
of recoveries is much higher because of the relevance of patients with mild symptoms which were not tested. However, the model simulated number of total recovered patients at the end of May 2020 is of about half a million, which five per cent of the total population in Lombardy.

Finally Figure 7.1 and 7.2 report the pattern of model based exposed and observed exposed (total cases- death-fatalities), looking at their level and their daily changes.

**INSERT FIGURE 7.1-7.2 HERE**

The figure shows that the observed daily exposed are very much in line with the model prediction, taking into account the fact that while Susceptible becomes Exposed instantaneously in the model, their observation in the data requires testing which is implemented only some time after hospitalization. The model based pattern of the change in Exposed which again is followed with a lag by the actual data might be of help in designing optimal interpolants for reduced form data based prediction of the dynamics of the virus.(Peracchi(2020))

## 5 Conclusions and Policy Implications

A SEIRCH Model that endogeneizes the fatality of the COVID disease in Lombardy, by taking into account the effect of the constraint in ICU positions is capable of explaining the very high COVID-19 related mortality in Lombardy while keeping the CFR of COVID at the value of 1.38 per cent.

The model also shows that the number of actual recovered patients in Lombardy is much higher than the number of patients recovered from hospitalization but to an order of magnitude that implies that by the end of May only 5 per cent of the Lombardy population will be recovered and therefore immune.

The disalignment between observed fatality and the standrd SEIR model simulated fatality in Lombardy can be entirely explained by the importance of the ICU constraint.

This is a very relevant element to keep monitored in the design of exit strategies from the lockdown.

The importance of the economic benefit of the number recovered patients should be carefully weighted against the potential high cost in human lives that will be incurred in whenever exit strategies from the lockdown cause an increase of $R_0$ that can make the ICU constraint binding again.
6 Appendix: The SEIRHC Model Specification

We report in this appendix the full model specification equation by equation. The model is made of 16 equations. It allows to simulate the dynamics of the virus diffusion starting from an initial period in which the total Population \((N_t)\) of \(N\) individuals is divided in 1 Infectious \((I_t)\) and \(N-1\) Susceptible \((S_t)\). In each period (day) some Susceptible become Exposed \((E_t)\), their number is determined by the basic reproduction number \(R_0\), that determines the number of secondary infections each infected individual produces, by the probability with which Susceptible meets Infectious, \((\frac{I_{t-1}}{N_{t-1}})\), and by the average duration of the period in which a patient is infectious \(T_{inf}\). Exposed after an incubation period of length \(T_{inc}\), become Infectious. The outflows from Susceptible is the inflows into Exposed in each period, and the outflows from Exposed is the inflows into Infectious. Infectious falls into three groups: those with mild symptoms \((MILD_t)\), those with severe symptoms \((SEV_t)\), and those with fatal symptoms \((FAT_t)\). The allocation to these groups is controlled by three probabilities: \((1 - p^{sev} - p^{fat})\), \(p^{sev}\), \(p^{fat}\). Patients with mild symptoms recover after a recovery period, \(T_{srec}\). The daily change in Mild patients stock is determined by the share \((1 - p^{sev} - p^{fat})\) of the outflows from Infectious, and the outflows from the share of Mild who recover that depends on the average duration form symptoms to recovery for mild patients, \(T_{srec}\). Patiens with severe and fatal symptoms require hospitalization, both these group are hospitalized after a period between developing symptoms and hospitalization of average duration \(T_{shosp}\), hospitalized. The daily change in Fatal patients is determined by the share \(p^{fat}\) of the outflows from Infectious and the outflows by the share of Fatal who are hospitalized. Patient with fatal symptoms succumb notwithstanding hospitalization, even in intensive care, after the mean duration from the onset of symptoms to death, \(T_{sd}\). The daily change in Severe patients is determined by the share \(p^{sev}\) of the outflows from Infectious and the outflows by the share of Severe who are hospitalized. Patients in hospital, independently from their initial status, require intensive care \(p^{sev\ unit}\) with probability \(p^{ic}\). Patient with severe symptoms either recover or become fatal. The recovered, with a mean duration of from the onset of symptoms to hospital discharge of \(T_{shi}\), are those who do need intensive care unit and those who need intensive care unit and find a place. The patients with severe symptoms that need ICU and do not find availability become fatal. At the end of each period the population decreases because of the fatalities, while the stock of recovered grows as a consequence from the new additions of recovered with mild and severe symptoms.
\[
\begin{align*}
\Delta S_t &= \left( \frac{R_0}{T_{inf} N_{t-1}} \right) S_{t-1} \\
\Delta E_t &= \left( \frac{R_0}{T_{inf} N_{t-1}} \right) S_{t-1} - \left( \frac{1}{T_{inc}} \right) E_{t-1} \\
\Delta I_t &= \left( \frac{1}{T_{inc}} \right) E_{t-1} - \left( \frac{1}{T_{inf}} \right) I_{t-1} \\
\Delta MILD_t &= p^{mild} \left( \frac{1}{T_{inf}} \right) I_{t-1} - \left( \frac{1}{T_{srec}} \right) MILD_{t-1} \\
\Delta REC\_MILD_t &= \left( \frac{1}{T_{srec}} \right) MILD_{t-1} \\
\Delta SEV_t &= p^{sev} \left( \frac{1}{T_{inf}} \right) I_{t-1} - \left( \frac{1}{T_{shosp}} \right) SEV_{t-1} \\
\Delta SEV\_H_t &= \left( \frac{1}{T_{shosp}} \right) SEV_{t-1} - \left( \frac{1}{T_{shd} - T_{inf}} \right) SEV\_H_{t-1} \\
\Delta SEV\_FAT_t &= I^{ICCS}_t \left( p^{ic\_SEV\_H_t} + p^{ic\_FAT\_H_t} - ICC_t \right) \\
\Delta REC\_SEV_t &= \left( \frac{1}{T_{shd} - T_{inf}} \right) SEV\_H_{t-1} - \Delta SEV\_FAT_t \\
\Delta FAT_t &= p^{fat} \left( \frac{1}{T_{inf}} \right) I_{t-1} - \left( \frac{1}{T_{shosp}} \right) FAT_{t-1} \\
\Delta FAT\_H_t &= \left( \frac{1}{T_{shosp}} \right) FAT_{t-1} - \left( \frac{1}{T_{sd} - T_{shosp}} \right) FAT\_H_{t-1} \\
\Delta REM\_FAT_t &= \left( \frac{1}{T_{sd} - T_{shosp}} \right) FAT\_H_{t-1} \\
HOSPITALIZED_t &= SEV\_H_t + FAT\_H_t \\
EFF\_FAT_t &= REM\_FAT_t + SEV\_FAT_t \\
RECOVERED_t &= REC\_MILD_t + REC\_SEV_t \\
\Delta N_t &= - \Delta EFF\_FAT_t \\
I^{ICCS}_t &= \begin{cases} 
1 & \text{if } \left( p^{ic\_SEV\_H_t} + p^{ic\_FAT\_H_t} > ICC_t \right) \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]
## Table and Figures

### Table 1: Calibration of the SEIRHC model for Lombardy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pre-lockdown</th>
<th>Post-Lockdown</th>
<th>Source PreL</th>
<th>Source PostL</th>
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<tbody>
<tr>
<td>$R_0$</td>
<td>2.2</td>
<td>0.95</td>
<td>EC</td>
<td>Calibrated$^1$</td>
</tr>
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<td>$T_{inf}$</td>
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<td>2.9 days</td>
<td>EC</td>
<td>EC</td>
</tr>
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<td>$T_{inc}$</td>
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<td>5.2 days</td>
<td>EC</td>
<td>EC</td>
</tr>
<tr>
<td>$T_{srec}$</td>
<td>11.1 days</td>
<td>11.1 days</td>
<td>EC</td>
<td>EC</td>
</tr>
<tr>
<td>$T_{shosp}$</td>
<td>5 days</td>
<td>5 days</td>
<td>EC</td>
<td>EC</td>
</tr>
<tr>
<td>$T_{sd}$</td>
<td>17.8 days</td>
<td>17.8 days</td>
<td>Verity et al.</td>
<td>Verity et al.</td>
</tr>
<tr>
<td>$T_{shd}$</td>
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<td>22.6 days</td>
<td>Verity et al.</td>
<td>Verity et al.</td>
</tr>
<tr>
<td>$p_{fat}$</td>
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<td>0.0138</td>
<td>Verity et al.</td>
<td>Verity et al.</td>
</tr>
<tr>
<td>$p_{sev}$</td>
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<td>0.1</td>
<td>IE</td>
<td>IE</td>
</tr>
<tr>
<td>$p_{ic}$</td>
<td>0.15</td>
<td>0.15</td>
<td>PC</td>
<td>PC</td>
</tr>
</tbody>
</table>

IC (International Evidence) https://www.worldometers.info/coronavirus/#countries
PC (Protezione Civile data on Lombardy) https://github.com/pcm-dpc/COVID-19

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$^1$The value is chosen to generate a match between model simulated hospitalization and observed hospitalization in Lombardy two weeks after the lockdown date.
Figure 1: COVID observed fatality rate, intensity of ICU usage in Lombardy, and SEIR model simulated fatality rate (daily data February 24th-March 31st)

Figure 2: COVID in Lombardy. Daily Fatalities and Patients in ICU
Figure 3: The SEIRHC model dependency graph
Figure 4: Actual and Simulated COVID Hospitalization in Lombardy

Figure 5: Actual and Simulated Fatalities in Lombardy
Figure 6: Model Simulated Recovered patients and observed recovered (guariti)
Figure 7.1: model simulated and observed exposed

Figure 7.2: Model Simulated and Observed Change in Exposed
8 References


Cereda et al. (2020) "The early phase of COVID-19 outbreak in Lombardy"


